# New Application of the $(G'/G)$-Expansion Method to Excite Soliton Structures for Nonlinear Equation

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The $(G'/G)$-expansion method is extended to construct non-travelling wave solutions for high-dimensional nonlinear equations and to explore special soliton structure excitations and evolutions. Taking an example, a new series of the non-travelling wave solutions are calculated for the (2+1)-dimensional asymmetrical Nizhnik-Novikov-Veselov system by using the $(G'/G)$-expansion method. By selecting appropriately the arbitrary functions in the solutions, special soliton-structure excitations and evolutions are studied.

*Key words:* $(G'/G)$-Expansion Method; (2+1)-Dimensional Asymmetrical Nihnik-Novikov-Veselov System; Non-Travelling Wave Solution; Soliton Structure Excitation.
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## 1. Introduction

The dynamics of soliton structures is a fascinating subject for nonlinear evolution equations (NEEs) which are drawn from interesting nonlinear physical phenomena. The soliton structures can be considered as a type of the special exact solutions of NEEs. It is a quite difficult but significant task to generate the localized soliton structures for (2+1) (2 spatial and 1 temporal)-dimensional NEEs. Just a few approaches have been mentioned in the literature, such as the variable separation method [1 – 11], the Riccati equation mapping method [12 – 23], and the similarity reduction method [24].

Very recently, an approach called the $(G'/G)$-expansion method was proposed to obtain new exact solutions of NEEs [25]. Subsequently the powerful $(G'/G)$-expansion method has been widely used by many such as in [26 – 34]. The method bases on the homogeneous balance principle and the linear ordinary differential equation (LODE) theory. In general, the solutions obtained by the $(G'/G)$-expansion method include three types, namely, hyperbolic function solutions, trigonometric function solution, and rational function solutions. However, the previous works have mainly concentrated on obtaining new exact travelling wave solutions for NEEs.

Our goal of this paper is to extend the $(G'/G)$-expansion method to construct non-travelling wave solutions with arbitrary functions, which can be used as seed functions to excite localized soliton structures for high-dimensional NEEs. As one application of the method, we will consider the well-known (2+1)-dimensional asymmetrical Nizhnik-Novikov-Veselov (ANNV) system

$$u_t + u_{xxx} - 3v_x u - 3v u_x = 0, \quad (1)$$

$$u_x + v_y = 0. \quad (2)$$

The ANNV system (1) – (2) was initially introduced by Boiti et al. [35], and its several soliton structures were studied respectively by the variable separation method [36] and the Riccati equation mapping method [37].

The organization of this paper is as follows. Section 2 is devoted to describe the process of constructing a new series of the non-travelling wave solutions of the ANNV system by extending the $(G'/G)$-expansion method. On the basis of the solutions, a type of the localized soliton structure excitation can be studied through appropriate selection of the arbitrary functions in Section 3. Finally, we conclude in Section 4.

## 2. The $(G'/G)$-expansion Method and the Non-Travelling Wave Solutions of the ANNV System

For a given (2+1)-dimensional NEE with independent variables $x,y,t$ and dependent variable $u$

$$F(u,u_t,u_x,u_y,u_{tt},u_{xt},u_{yt},u_{xy},u_{xx},u_{yy},\ldots)=0, \quad (3)$$

the fundamental idea of the $(G'/G)$-expansion method is that the solutions of (3) can be expressed by a polynomial in $(G'/G)$ as follows [25]:

$$u=\sum_{i=0}^{n} a_i\left[\frac{G'(q)}{G(q)}\right], \quad (4)$$

where $q=sx+ly-Vt$ is the travelling wave transformation, and $s,l,V$, $a_i$ $(i=0,1,2,\ldots,n)$ are constants to be determined later, $G(q)$ satisfies the second order LODE as follows:

$$G''+\lambda G'+\mu G=0. \quad (5)$$

In order to construct the non-travelling wave solutions with arbitrary function $q(x,y,t)$ for the ANNV system (1)–(2), we suppose its solutions can be express as follows:

$$u=\sum_{i=0}^{n} a_i\left[\frac{G'(q)}{G(q)}\right]^i + \sum_{j=1}^{n} A_j\left[\frac{G'(q)}{G(q)}\right]^{j-1}\left\{-\left[\frac{G'(q)}{G(q)}\right]'\right\}^{\frac{1}{2}}, \quad (6)$$

$$v=\sum_{i=0}^{m} b_i\left[\frac{G'(q)}{G(q)}\right]^i + \sum_{j=1}^{m} B_j\left[\frac{G'(q)}{G(q)}\right]^{j-1}\left\{-\left[\frac{G'(q)}{G(q)}\right]'\right\}^{\frac{1}{2}}, \quad (7)$$

where $a_i$ $(i=0,1,2,\ldots,n)$, $A_j$ $(j=1,2,\ldots,n)$, $b_i$ $(i=0,1,2,\ldots,m)$, $B_j$ $(j=1,2,\ldots,m)$, are functions of $x,y,t$ to be determined later, $q=q(x,y,t)$ is an arbitrary function of $x,y,t$, and $G(q)$ satisfies the second-order LODE as follows:

$$G''+\mu G=0. \quad (8)$$

**Remark 1:** Compared with (4), the travelling wave transformation $q=sx+ly-Vt$ is a special case of the arbitrary function $q$ of $x,y,t$ in (6) and (7). Thus the coefficient $a_i,A_j,b_i,B_j$ in (6) and (7) must be functions of $x,y,t$.

**Remark 2:** In (6) and (7), the added term of $[G'(q)/G(q)]^{j-1}\left\{-[G'(q)/G(q)]'\right\}^{1/2}$ is for obtaining more helpful equations to solve $a_i$, $A_j$, $b_i$, $B_j$.

Applying the homogenous balance principle [38], we obtain $n=m=2$. Thus (6)–(7) can be converted into

$$u=a_0+a_1\left(\frac{G'}{G}\right)+a_2\left(\frac{G'}{G}\right)^2+A_1\left[-\left(\frac{G'}{G}\right)'\right]^{\frac{1}{2}} + A_2\left(\frac{G'}{G}\right)\left[-\left(\frac{G'}{G}\right)'\right]^{\frac{1}{2}}, \quad (9)$$

$$v=b_0+b_1\left(\frac{G'}{G}\right)+b_2\left(\frac{G'}{G}\right)^2+B_1\left[-\left(\frac{G'}{G}\right)'\right]^{\frac{1}{2}} + B_2\left(\frac{G'}{G}\right)\left[-\left(\frac{G'}{G}\right)'\right]^{\frac{1}{2}}, \quad (10)$$

where $a_0,a_1,a_2,A_1,A_2,b_0,b_1,b_2,B_1,B_2$ are functions of $x,y,t$ to be determined later, and $q$ is an arbitrary function of $x$, $y$, $t$.

For simplifying the computation, we seek for the variable separation solutions of the ANNV system (1)–(2) by taking $q(x,y,t)=f(x,t)+g(y,t)$.

Substituting (9)–(10) into the ANNV system (1)–(2), collecting all terms with the same power of $(G'/G)$ together, the left-hand sides of the ANNV system (1)–(2) are converted into the polynomials in $(G'/G)$. Then setting each coefficient of the polynomials to zero, we can derive a set of over-determined partial differential equation for $a_0$, $a_1$, $a_2$, $A_1$, $A_2$, $b_0$, $b_1$, $b_2$, $B_1$, $B_2$, and $q$.

$$\left(\frac{G'}{G}\right)^5: 2a_2q_x^2-a_2b_2-A_2B_2=0, \quad (11)$$

$$\left(\frac{G'}{G}\right)^4\left[-\left(\frac{G'}{G}\right)'\right]^{\frac{1}{2}}: 2A_2q_x^2-a_2B_2-A_2b_2=0, \quad (12)$$

$$\left(\frac{G'}{G}\right)^4: 3(a_1b_2+a_2b_1+A_2B_1+A_1B_2) + 2[(a_2q_x)_x+q_x(a_{2x}-a_1q_x)]=0, \quad (13)$$

$$\left(\frac{G'}{G}\right)^3\left[-\left(\frac{G'}{G}\right)'\right]^{\frac{1}{2}}: 3(a_1B_2+A_2b_1+a_2B_1 + A_1b_2)+2\left[(A_2q_x)_x+q_x(A_{2x}-A_1q_x)\right]=0, \quad (14)$$

$$\left(\frac{G'}{G}\right)^3 : 3q_x(a_0b_2+a_2b_0+a_1b_1+A_1B_1+\mu A_2B_2)-a_2q_t-q_x(a_{2x}-a_1q_x)_x+a_{1x}q_x^2-8\mu a_2q_x^3=0, \quad (15)$$

$$\left(\frac{G'}{G}\right)^2\left[-\left(\frac{G'}{G}\right)'\right]^{\frac{1}{2}} : 3q_x(a_1B_1+A_1b_1+a_0B_2+A_2b_0)-A_2q_t-q_x(A_{2x}-A_1q_x)_x + q_x^2(A_{1x}-\mu A_2q_x)-4\mu A_2q_x^3=0, \quad (16)$$

$$\left(\frac{G'}{G}\right)^2 : 3q_x(a_0b_1+a_1b_0+\mu A_2B_1+\mu A_1B_2)-a_1q_t-q_x(a_{1x}-2\mu a_2q_x)_x+2\mu q_x^2(a_{2x}-a_1q_x)=0, \quad (17)$$

$$\left(\frac{G'}{G}\right)\left[-\left(\frac{G'}{G}\right)'\right]^{\frac{1}{2}} : 3q_x(a_0B_1+A_1b_0)-A_1q_t-q_x(A_{1x}-\mu A_2q_x)_x+\mu q_x^2(A_{2x}-A_1q_x)=0, \quad (18)$$

$$\left(\frac{G'}{G}\right) : 3(a_0b_1+a_1b_0+\mu A_2B_1+\mu A_1B_2)_x-a_{1t}-(a_{1x}-2\mu a_2q_x)_{xx}+2\mu[q_x(a_{2x}-a_1q_x)]_x=0, \quad (19)$$

$$\left(\frac{G'}{G}\right)^0\left[-\left(\frac{G'}{G}\right)'\right]^{\frac{1}{2}} : 3(a_0B_1+A_1b_0)_x-A_{1t}-(A_{1x}-\mu A_2q_x)_{xx}+\mu[q_x(A_{2x}-A_1q_x)]_x=0, \quad (20)$$

$$\left(\frac{G'}{G}\right)^0 : 3(a_0b_0+\mu A_1B_1)_x-a_{0t}-(a_{0x}-\mu a_1q_x)_{xx}+\mu[q_x(a_{1x}-2\mu a_2q_x)]_x=0, \quad (21)$$

$$\left(\frac{G'}{G}\right)^3 : a_2q_x-b_2q_y=0, \quad (22)$$

$$\left(\frac{G'}{G}\right)^2\left[-\left(\frac{G'}{G}\right)'\right]^{\frac{1}{2}} : A_2q_x-B_2q_y=0, \quad (23)$$

$$\left(\frac{G'}{G}\right)^2 : a_{2x}-a_1q_x-b_{2y}+b_1q_y=0, \quad (24)$$

$$\left(\frac{G'}{G}\right)\left[-\left(\frac{G'}{G}\right)'\right]^{\frac{1}{2}} : A_{2x}-A_1q_x-B_{2y}+B_1q_y=0, \quad (25)$$

$$\left(\frac{G'}{G}\right) : a_{1x}-b_{1y}=0, \quad (26)$$

$$\left(\frac{G'}{G}\right)^0\left[-\left(\frac{G'}{G}\right)'\right]^{\frac{1}{2}} : A_{1x}-B_{1y}=0, \quad (27)$$

$$\left(\frac{G'}{G}\right)^0 : a_{0x}-\mu a_1q_x-b_{0y}+\mu b_1q_y=0. \quad (28)$$

Solving (11) – (28) yields

$$\begin{aligned} &a_0=\mu q_xq_y, \quad b_0=\frac{q_t+q_{xxx}+2\mu q_x^3}{3q_x},\\ &a_1=A_1=0, \quad a_2=-A_2=q_xq_y,\\ &b_2=-B_2=q_x^2, \quad B_1=-b_1=q_{xx}, \end{aligned} \quad (29)$$

and

$$\begin{aligned} &a_0=\mu q_xq_y, \quad b_0=\frac{q_t+q_{xxx}+2\mu q_x^3}{3q_x},\\ &a_1=A_1=0, \quad a_2=A_2=q_xq_y,\\ &b_2=B_2=q_x^2, \quad B_1=b_1=-q_{xx}, \end{aligned} \quad (30)$$

Substituting (29), (30), and the general solutions of (8) into (9) – (10), we can obtain the non-travelling wave solutions for the ANNV system (1) – (2).

**Case 1.** When $\mu<0$, the hyperbolic function solutions for the ANNV system (1) – (2) are the following:

$$\begin{aligned} u_1=&\mu f_xg_y-\mu f_xg_y\left(\frac{C_1\sinh\sqrt{-\mu}q+C_2\cosh\sqrt{-\mu}q}{C_1\cosh\sqrt{-\mu}q+C_2\sinh\sqrt{-\mu}q}\right)^2\\ &+\mu f_xg_y\frac{C_1\sinh\sqrt{-\mu}q+C_2\cosh\sqrt{-\mu}q}{C_1\cosh\sqrt{-\mu}q+C_2\sinh\sqrt{-\mu}q}\\ &\cdot\left[\frac{C_2^2-C_1^2}{(C_1\cosh\sqrt{-\mu}q+C_2\sinh\sqrt{-\mu}q)^2}\right]^{\frac{1}{2}}, \end{aligned} \quad (31)$$

$$\begin{aligned} v_1=&\frac{f_t+g_t+f_{xxx}+2\mu f_x^3}{3f_x}\\ &-\sqrt{-\mu}f_{xx}\frac{C_1\sinh\sqrt{-\mu}q+C_2\cosh\sqrt{-\mu}q}{C_1\cosh\sqrt{-\mu}q+C_2\sinh\sqrt{-\mu}q}\\ &-\mu f_x^2\left(\frac{C_1\sinh\sqrt{-\mu}q+C_2\cosh\sqrt{-\mu}q}{C_1\cosh\sqrt{-\mu}q+C_2\sinh\sqrt{-\mu}q}\right)^2\\ &+\sqrt{-\mu}f_{xx}\left[\frac{C_2^2-C_1^2}{(C_1\cosh\sqrt{-\mu}q+C_2\sinh\sqrt{-\mu}q)^2}\right]^{\frac{1}{2}}\\ &+\mu f_x^2\frac{C_1\sinh\sqrt{-\mu}q+C_2\cosh\sqrt{-\mu}q}{C_1\cosh\sqrt{-\mu}q+C_2\sinh\sqrt{-\mu}q}\\ &\cdot\left[\frac{C_2^2-C_1^2}{(C_1\cosh\sqrt{-\mu}q+C_2\sinh\sqrt{-\mu}q)^2}\right]^{\frac{1}{2}}, \end{aligned} \quad (32)$$

and

$$u_2 = \mu f_x g_y - \mu f_x g_y \left(\frac{C_1 \sinh\sqrt{-\mu}q + C_2 \cosh\sqrt{-\mu}q}{C_1 \cosh\sqrt{-\mu}q + C_2 \sinh\sqrt{-\mu}q}\right)^2 - \mu f_x g_y \frac{C_1 \sinh\sqrt{-\mu}q + C_2 \cosh\sqrt{-\mu}q}{C_1 \cosh\sqrt{-\mu}q + C_2 \sinh\sqrt{-\mu}q}\left[\frac{C_2^2 - C_1^2}{(C_1 \cosh\sqrt{-\mu}q + C_2 \sinh\sqrt{-\mu}q)^2}\right]^{\frac{1}{2}}, \tag{33}$$

$$v_2 = \frac{f_t + g_t + f_{xxx} + 2\mu f_x^3}{3 f_x} - \sqrt{-\mu} f_{xx} \frac{C_1 \sinh\sqrt{-\mu}q + C_2 \cosh\sqrt{-\mu}q}{C_1 \cosh\sqrt{-\mu}q + C_2 \sinh\sqrt{-\mu}q} - \mu f_x^2 \left(\frac{C_1 \sinh\sqrt{-\mu}q + C_2 \cosh\sqrt{-\mu}q}{C_1 \cosh\sqrt{-\mu}q + C_2 \sinh\sqrt{-\mu}q}\right)^2 - \sqrt{-\mu} f_{xx}\left[\frac{C_2^2 - C_1^2}{(C_1 \cosh\sqrt{-\mu}q + C_2 \sinh\sqrt{-\mu}q)^2}\right]^{\frac{1}{2}} - \mu f_x^2 \frac{C_1 \sinh\sqrt{-\mu}q + C_2 \cosh\sqrt{-\mu}q}{C_1 \cosh\sqrt{-\mu}q + C_2 \sinh\sqrt{-\mu}q}\left[\frac{C_2^2 - C_1^2}{(C_1 \cosh\sqrt{-\mu}q + C_2 \sinh\sqrt{-\mu}q)^2}\right]^{\frac{1}{2}}, \tag{34}$$

where $q = f(x,t) + g(y,t)$, $f_x \neq 0$, $f_{xxx}$ and $g_y, g_t$ exist, and $C_2^2 - C_1^2 \geq 0$.

**Case 2.** When $\mu > 0$, the trigonometric function solutions for the ANNV system (1)–(2) are the following:

$$u_3 = \mu f_x g_y + \mu f_x g_y \left(\frac{-C_1 \sin\sqrt{\mu}q + C_2 \cos\sqrt{\mu}q}{C_1 \cos\sqrt{\mu}q + C_2 \sin\sqrt{\mu}q}\right)^2 - \mu f_x g_y \frac{-C_1 \sin\sqrt{\mu}q + C_2 \cos\sqrt{\mu}q}{C_1 \cos\sqrt{\mu}q + C_2 \sin\sqrt{\mu}q}\left[\frac{C_1^2 + C_2^2}{(C_1 \cos\sqrt{\mu}q + C_2 \sin\sqrt{\mu}q)^2}\right]^{\frac{1}{2}}, \tag{35}$$

$$v_3 = \frac{f_t + g_t + f_{xxx} + 2\mu f_x^3}{3 f_x} - \sqrt{\mu} f_{xx} \frac{-C_1 \sin\sqrt{\mu}q + C_2 \cos\sqrt{\mu}q}{C_1 \cos\sqrt{\mu}q + C_2 \sin\sqrt{\mu}q} + \mu f_x^2 \left(\frac{-C_1 \sin\sqrt{\mu}q + C_2 \cos\sqrt{\mu}q}{C_1 \cos\sqrt{\mu}q + C_2 \sin\sqrt{\mu}q}\right)^2 + \sqrt{\mu} f_{xx}\left[\frac{C_1^2 + C_2^2}{(C_1 \cos\sqrt{\mu}q + C_2 \sin\sqrt{\mu}q)^2}\right]^{\frac{1}{2}} - \mu f_x^2 \frac{-C_1 \sin\sqrt{\mu}q + C_2 \cos\sqrt{\mu}q}{C_1 \cos\sqrt{\mu}q + C_2 \sin\sqrt{\mu}q}\left[\frac{C_1^2 + C_2^2}{(C_1 \cos\sqrt{\mu}q + C_2 \sin\sqrt{\mu}q)^2}\right]^{\frac{1}{2}}, \tag{36}$$

and

$$u_4 = \mu f_x g_y + \mu f_x g_y \left(\frac{-C_1 \sin\sqrt{\mu}q + C_2 \cos\sqrt{\mu}q}{C_1 \cos\sqrt{\mu}q + C_2 \sin\sqrt{\mu}q}\right)^2 + \mu f_x g_y \frac{-C_1 \sin\sqrt{\mu}q + C_2 \cos\sqrt{\mu}q}{C_1 \cos\sqrt{\mu}q + C_2 \sin\sqrt{\mu}q}\left[\frac{C_1^2 + C_2^2}{(C_1 \cos\sqrt{\mu}q + C_2 \sin\sqrt{\mu}q)^2}\right]^{\frac{1}{2}}, \tag{37}$$

$$v_4 = \frac{f_t + g_t + f_{xxx} + 2\mu f_x^3}{3 f_x} - \sqrt{\mu} f_{xx} \frac{-C_1 \sin\sqrt{\mu}q + C_2 \cos\sqrt{\mu}q}{C_1 \cos\sqrt{\mu}q + C_2 \sin\sqrt{\mu}q} + \mu f_x^2 \left(\frac{-C_1 \sin\sqrt{\mu}q + C_2 \cos\sqrt{\mu}q}{C_1 \cos\sqrt{\mu}q + C_2 \sin\sqrt{\mu}q}\right)^2 - \sqrt{\mu} f_{xx}\left[\frac{C_1^2 + C_2^2}{(C_1 \cos\sqrt{\mu}q + C_2 \sin\sqrt{\mu}q)^2}\right]^{\frac{1}{2}} + \mu f_x^2 \frac{-C_1 \sin\sqrt{\mu}q + C_2 \cos\sqrt{\mu}q}{C_1 \cos\sqrt{\mu}q + C_2 \sin\sqrt{\mu}q}\left[\frac{C_1^2 + C_2^2}{(C_1 \cos\sqrt{\mu}q + C_2 \sin\sqrt{\mu}q)^2}\right]^{\frac{1}{2}}, \tag{38}$$

where $q = f(x,t) + g(y,t)$, $f_x \neq 0$, $f_{xxx}$ and $g_y, g_t$ exist, and $C_1^2 + C_2^2 \neq 0$, namely, $C_1 \neq 0$ and $C_2 \neq 0$.

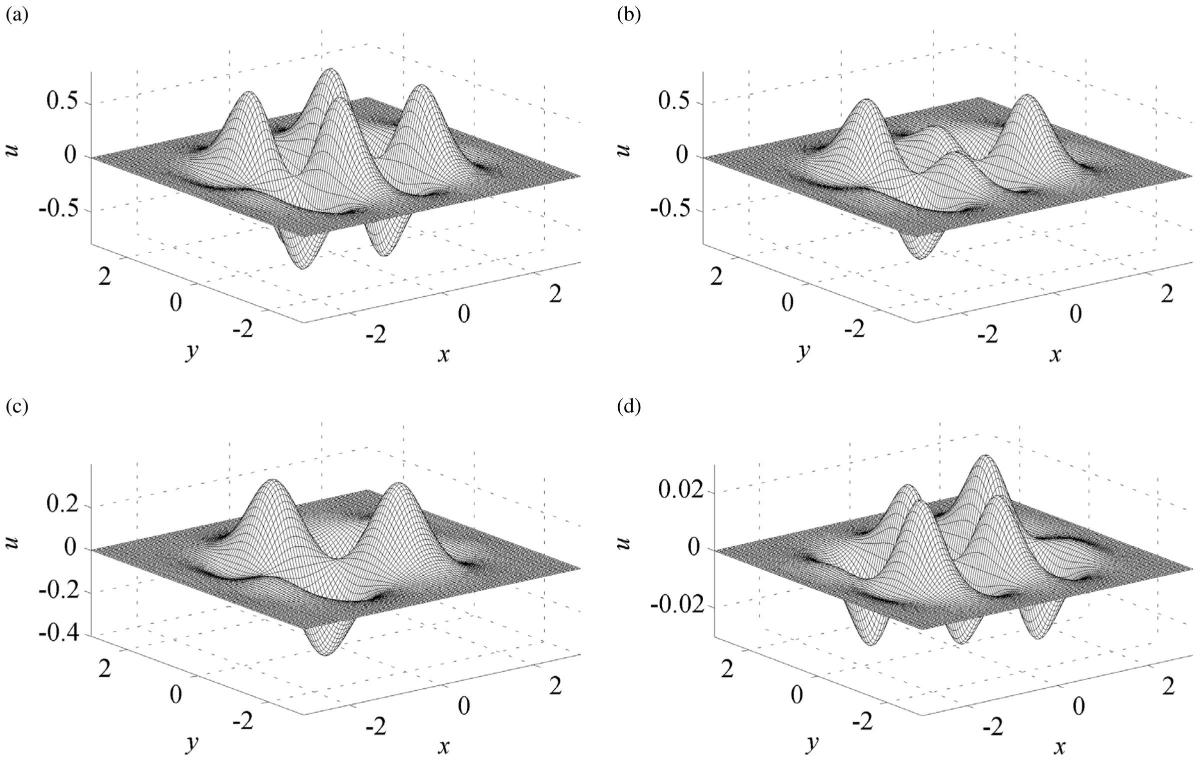


Fig. 1. Evolution plots of the solution (31) under the parameters (42) with time: (a) $t = 0$; (b) $t = 0.6$; (c) $t = 1.2$; (d) $t = 1.8$.

**Case 3.** When $\mu = 0$, the rational function solution for the ANNV system (1) – (2) is the following:

$$u_5 = 2f_x g_y \left(\frac{C_1}{C_1 q + C_2}\right)^2, \tag{39}$$

$$v_5 = \frac{f_t + g_t + f_{xxx}}{3f_x} + 2\left(\frac{C_1}{C_1 q + C_2}\right)\left[f_x^2\left(\frac{C_1}{C_1 q + C_2}\right) - f_{xx}\right], \tag{40}$$

where $q = f(x,t) + g(y,t)$, $f_x \neq 0$, $f_{xxx}$ and $g_y, g_t$ exist, and $C_1 \neq 0$.

Comparing the solutions (31) – (40) with those in [36] obtained by the variable separation method, we find that there are some connections between these solutions. If setting $C_1 = 1$, $C_2 = 0$, the solutions (39) and (40) can be degenerated to the solutions (19) and (20) given in [36], respectively.

## 3. Soliton Structure Excitation

Thanks to the arbitrary functions $f(x,t)$ and $g(y,t)$ in the solutions (31) – (40), it is convenient to excite abundant soliton structures. We take the solution (31) as an example to study the soliton excitations for the ANNV system (1) – (2). For instance, we select

$$f(x,t) = \sin(-x^2 - t^2)\exp(-x^2 - t^2), \quad g(y,t) = \sin(-y^2)\exp(-y^2). \tag{41}$$

Substituting (41) into (31) leads to a type of oscillatory dromion soliton structure for the ANNV system (1) – (2). Figure 1a to d are the evolution plots of the solution (31) with time under the parameters

$$C_1 = 1,\ C_2 = 1.2,\ \mu = -1. \tag{42}$$

Figure 2a – d are the corresponding projection plots to Figure 1a – d on the $x,u$ plane. The amplitude of the soliton structure varies dramatically while the wave shape fluctuates with time.

Above we showed the excitation process of a special dromion soliton structure of the solution (31) for the ANNV system (1) – (2). It is clear that other selections of the arbitrary functions $f(x,t)$ and $g(y)$ in (31) may generate rich localized soliton structures. On the other hand, the solutions (31) – (40) may also be used to excite abundant soliton structures.

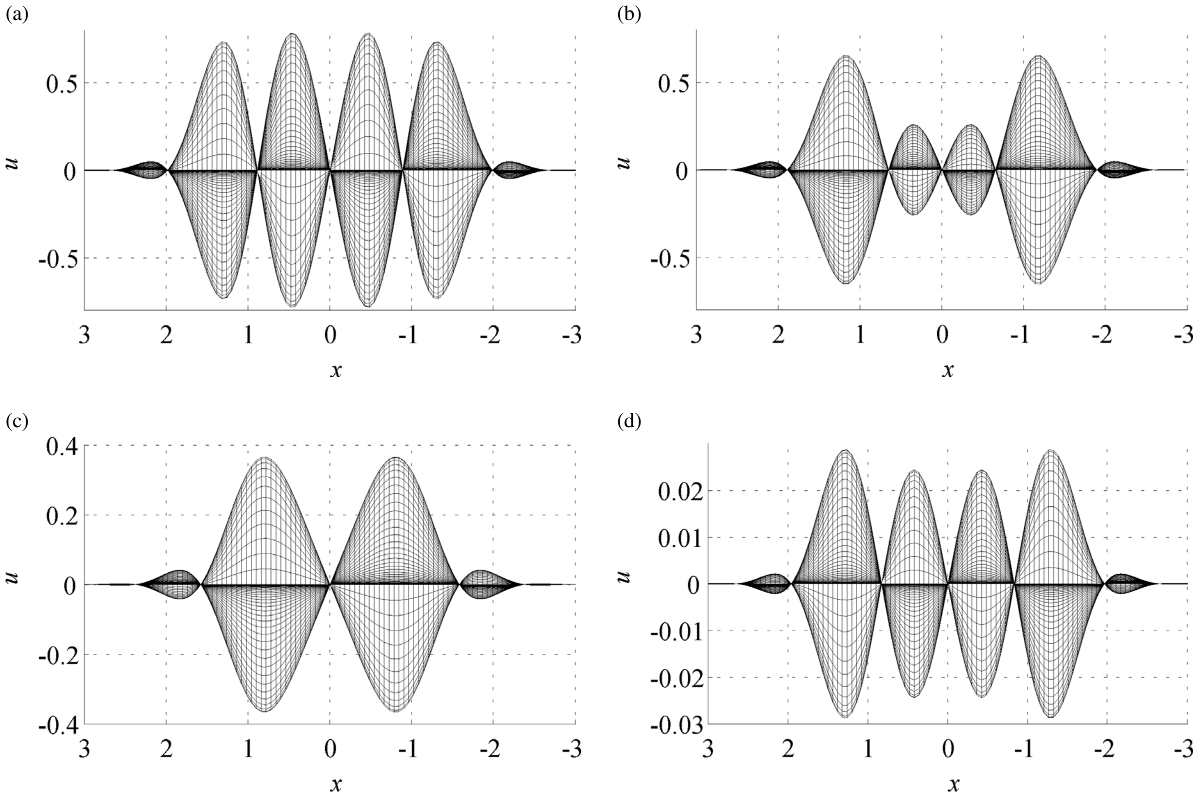


Fig. 2. Corresponding projective plots to Figure 1 on the $x,u$ plane: (a) $t = 0$; (b) $t = 0.6$; (c) $t = 1.2$; (d) $t = 1.8$.

## 4. Conclusion

By extending the ($G'/G$)-expansion method, we are able to construct a new series of exact non-travelling wave solutions of the (2+1)-dimensional ANNV system. Furthermore, by selecting appropriately the arbitrary function $q(x,y,t)$ included in the solutions, one can study various interesting localized soliton excitations. Within our knowledge, it is the first attempt to explore the localized soliton excitations through the non-travelling wave solutions by the ($G'/G$)-expansion method. We believe the idea in this paper is also applied to other NEEs in the future.

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